

A DYNAMIC GENERAL LINEAR MODEL FOR INFERENCE FROM  
ACCELERATED LIFE TESTS

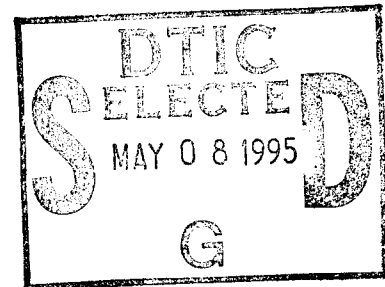
by

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# A DYNAMIC GENERAL LINEAR MODEL FOR INFERENCE FROM ACCELERATED LIFE TESTS

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## Abstract

We present a new approach for inference from accelerated life tests. Our approach is based on a dynamic general linear model setup which arises naturally from the accelerated life testing problem and uses linear Bayesian methods for inference. The advantage of the procedure is that it does not require large number of items to be tested and that it can deal with both censored and uncensored data. Furthermore, the approach produces closed form inference results. We illustrate the use of our approach with some actual accelerated life test data.

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## 1. INTRODUCTION AND OVERVIEW

In reliability studies it is a common practice to subject items to an environment which is more severe than the normal operating environment so that failures can be induced in a short amount of test time. A more severe environment can be created by increasing one or more of the stress levels, which constitute the environment, to values which are greater than their usual levels. Such tests are called *accelerated life tests*. The main problem with inference from accelerated life tests is that uncertainty statements about the failure behavior of the items at usual stress conditions have to be made using life length data from the more severe stress conditions. Most of the current literature on the accelerated life testing problem is based on the sample theoretic paradigm [see Meeker and Hahn (1985) for an up-to-date review]; exceptions to these are Meinhold and Singpurwalla (1984) and Blackwell and Singpurwalla (1986) which involve a *Kalman Filter* formulation of the accelerated life testing problem.

In this paper we present a new procedure for inference from accelerated life tests. Our procedure is Bayesian and is based on a *dynamic general linear model* (DGLM) setup [see West, Harrison and Migon (1985), henceforth WHM] which arises naturally from the accelerated life testing scenario. Our procedure uses the *linear Bayesian approach* of WHM and produces closed form solutions for inference. The main advantage of the procedure is that it does not require large number of items to be tested at each stress level and that it can deal with both censored and uncensored data. Besides, the recursive nature of the results produced by the procedure facilitates its use on Personal Computers.

In our setup we assume that lifetimes obtained under all stress levels have an exponential distribution and make use of a particular time transformation function. The extension of our approach to other failure distributions such as the Weibull distribution is being investigated. The extension of the procedure to other time transformation functions is straight forward.

A synopsis of our paper is as follows:

In Section 2 we present the notation and preliminaries for the accelerated life testing problem. In Section 3 we introduce the *power law* as a time transformation function and present the DGLM setup for the problem. We also discuss the adoption of our procedure to other time transformation functions. In Section 4 we describe the inference procedure for the DGLM setup. Finally in Section 5, we illustrate the use of our approach by applying it to some actual accelerated life testing data published by Nelson (1972).

## 2. NOTATION AND PRELIMINARIES

Assume that testing is done in stages using  $k$  accelerated stress levels of decreasing intensity which are specified in advance and may or may not be equally spaced. During the  $i$ -th stage of testing, items are tested under an accelerated stress level denoted by  $S_i$ . Using the notation  $S_i < S_j$  to denote that  $S_j$  is a more severe testing environment than  $S_i$ , we note that

$$S_1 > S_2 > \dots > S_k > S_{k+1}, \quad (2.1)$$

where  $S_{k+1}$  denotes the use stress at which no testing is done.

At each stress level  $S_i$ ,  $n_i$  items are tested for a predetermined and fixed time length  $\tau_i$ . Let  $Y_{ij}$  denote the time to failure of the  $j$ -th item tested under stress level  $S_i$ , and  $y_{ij}$  denote the realization of  $Y_{ij}$ , for  $j = 1, 2, \dots, r_i \leq n_i$  where  $r_i$  denotes the number of failures observed during the time interval  $(0, \tau_i]$ .

We assume that the failure rate function for items tested under stress  $S_i$  is constant and denoted by  $\lambda_i$ . Thus, given  $\lambda_i$  the failure distribution under stress  $S_i$  is described by the exponential density

$$p(y_i | \lambda_i) = \lambda_i e^{-\lambda_i y_i}. \quad (2.2)$$

Furthermore, given  $\lambda_i$ , failure times for items tested under stress  $S_i$  are judged to be independent.

Given the  $r_i$ 's and  $y_{ij}$ 's, our goal is to make inferences about the failure behavior of an item operating at use stress (normal) conditions  $S_{k+1}$ . In so doing, it is most common to assume a functional relationship between the failure rate and the applied stress level. Such relationship is known as an *acceleration* or *time transformation* function. Commonly used models for describing such relationship are the *Arrhenius Law*, the *Eyring Law*, and the *Power law*; see for example Mann, Schafer and Singpurwalla (1974), p. 421. In what follows, we will focus attention on the popular Power Law and present the DGLM setup for the accelerated life testing problem. In general, the use of any of these laws should be based on the physics of failure for the problem at hand.

### 3. THE DGLM SETUP FOR ACCELERATED LIFE TESTING

Under the Power Law, we write

$$\lambda_i = \alpha_i S_i^{\beta_i}, \quad (3.1)$$

where  $\alpha$  and  $\beta$  are unknown coefficients which describe the stress effect on failure rate. The subscripts associated with  $\alpha$  and  $\beta$  imply the fact that the time transformation function might be changing from one stress level to another. As noted by Blackwell and Singpurwalla (1986) this is likely to happen due to the changes in the basic failure mechanism with changes in the stress level.

We can linearize (3.1) by taking natural logarithms on both sides and write

$$\eta_i = \log \alpha_i + \beta_i \log S_i, \quad (3.2)$$

where  $\eta_i = \log \lambda_i$ . We define  $\underline{\theta}_i' = (\log \alpha_i \quad \beta_i)$ ,  $\underline{F}_i' = (1 \quad \log S_i)$  and write (3.2) as

$$\eta_i = \underline{F}_i' \underline{\theta}_i. \quad (3.3)$$

We note that (3.3) provides us with a *guide relationship* in the sense of WHM and that  $\underline{\theta}_i$  is the underlying state vector.

Next we describe how the time transformation function is changing from one stress level to another by specifying the *system (evolution)*

equation of the model as

$$\underline{\theta}_i = \underline{\theta}_{i-1} + \underline{w}_i, \quad (3.4)$$

where  $\underline{w}_i$  is a random innovation term. We note that in (3.4) the state vector  $\underline{\theta}_i$ 's are assumed to be constant from one stress level to another, except for the random changes brought about by the innovations  $\underline{w}_i$ . The distribution of  $\underline{w}_i$  is only partially specified through its first and second order moments. We use the notation,

$$\underline{w}_i \sim [ \underline{0}, \underline{W}_i ], \quad (3.5)$$

to denote the fact that  $\underline{w}_i$  has mean vector  $\underline{0}$  and variance-covariance matrix  $\underline{W}_i$ . Typically,  $\underline{w}_i$  is independent of  $\underline{\theta}_{i-1}$ .

Let  $D_i$  denote all the relevant information available at stage  $i$  after observing  $r_i$  and failure times  $\{y_{ij}\}_{j=1}^{r_i}$  at stress level  $S_i$ ;  $D_0$  represents all relevant information available at stage 0 prior to any testing. At stage  $(i-1)$ , we assume that the distribution of the state vector  $\underline{\theta}_{i-1}$  is partially described by the first and second order moments as

$$(\underline{\theta}_{i-1} | D_{i-1}) \sim [ \underline{m}_{i-1}, \underline{C}_{i-1} ]. \quad (3.6)$$

Using the system equation and (3.6) we can write

$$(\underline{\theta}_i | D_{i-1}) \sim [ \underline{m}_{i-1}, \underline{R}_i ], \quad (3.7)$$

where  $\underline{R}_i = \underline{C}_{i-1} + \underline{W}_i$ . Equation (3.7) provides us with a partial description of the prior distribution of the state vector  $\underline{\theta}_i$  before testing at stress level  $S_i$ . It is important to note that the innovation term  $\underline{w}_i$  provides an increase in uncertainty (represented by the addition of  $\underline{W}_i$ ) over the stress levels as  $\underline{\theta}_{i-1}$  changes to  $\underline{\theta}_i$ . This loss of information about  $\underline{\theta}_i$  motivates the *discount concept* [see for example, Smith (1979) and Ameen and Harrison(1985)] used by WHM as a guide for the choice of  $\underline{W}_i$ . The underlying idea is that the increase in uncertainty over the next stress level should be relative to that available at the present stress level, measured by  $\underline{C}_{i-1}$ . The details associated with the choice of the discount factor are described by WHM. In our case, the discount factor can be chosen as function of  $(S_i/S_{i-1})$  to take into account the relative magnitudes of the stresses.

From the guide relationship (3.3) we can write

$$f_i = E[\eta_i | D_{i-1}] = \underline{F}_i' \underline{m}_{i-1}, \quad (3.8)$$

$$q_i = \text{Var}[\eta_i | D_{i-1}] = \underline{F}_i' \underline{R}_i \underline{F}_i.$$

We note that (3.8) provides us with only the first two moments of the prior distribution of  $\eta_i$  at stage (i-1). We can specify a full distributional form for the prior of  $\eta_i$  and use the relationships given by (3.8) to determine the parameters of the distribution. In order to obtain closed form solutions in the Bayesian analysis, we will specify a conjugate form for the prior of  $\eta_i$

which is a loggamma density of the form

$$p(\eta_i | D_{i-1}) \propto \exp\{ a_i \eta_i - b_i e^{\eta_i} \}, \quad (3.9)$$

where  $a_i$  and  $b_i$  are the prior parameters. We denote the density in (3.9) by  $\mathcal{LG}(a_i, b_i)$ . From (3.9) we may obtain

$$E[\eta_i | D_{i-1}] = \Psi(a_i) - \log(b_i),$$

and

$$\text{Var}[\eta_i | D_{i-1}] = \Psi'(a_i),$$

where  $\Psi(\cdot)$  and  $\Psi'(\cdot)$  are the *digamma* and *trigamma* functions respectively.

We will specify the prior parameters  $a_i$  and  $b_i$  such that the first two moments of  $\eta_i$  agree with (3.8), that is,

$$\Psi(a_i) - \log(b_i) = r_i, \quad (3.10)$$

$$\Psi'(a_i) = q_i.$$

In solving (3.10) for  $a_i$  and  $b_i$  some approximations to  $\Psi(\cdot)$  and  $\Psi'(\cdot)$  can be used [see for example, Cox and Lewis (1966), Ch. 2]. We note that specifying the prior distribution of  $\eta_i$  as  $\mathcal{LG}(a_i, b_i)$  implies that the prior of  $\lambda_i$  at stage (i-1) is a gamma density with shape parameter  $a_i$  and scale parameter  $b_i$ . We will denote this density as  $\mathcal{G}(a_i, b_i)$ .

To summarize, the DGLM setup for the accelerated life testing problem is as follows:

$$\begin{aligned} p(y_i | \lambda_i) &= \lambda_i e^{-\lambda_i y_i}, \\ (\eta_i | D_{i-1}) &\sim \mathcal{LG}(a_i, b_i); \quad \eta_i = \log(\lambda_i), \\ (\underline{\theta}_i | D_{i-1}) &\sim [\underline{m}_{i-1}, \underline{R}_i] \end{aligned} \quad (3.11)$$

where  $a_i$  and  $b_i$  are chosen according to the guide relationship  $\eta_i = \underline{F}_i' \underline{\theta}_i$  so that  $E[\eta_i | D_{i-1}] = f_i$  and  $\text{Var}[\eta_i | D_{i-1}] = q_i$ .

We note that the DGLM setup for the accelerated life testing problem can be easily modified for other time transformation functions. For example, under the Arrhenius Law

$$\lambda_i = \exp\{\alpha_i - \beta_i / S_i\}, \quad (3.12)$$

which implies that the guidance relationship  $\eta_i = \log(\lambda_i)$  is given by equation (3.3) with  $\underline{\theta}' = (\alpha_i \quad \beta_i)$  and  $\underline{F}_i' = (1 \quad -1/S_i)$ . Now the DGLM setup of (3.11) can be applied to the problem. Extension to the Eyring and other laws follows along the same lines.

#### 4. INFERENCE RESULTS FOR DGLM SETUP

Given  $D_i$ , the available information after the  $i$ -th stage of testing, we need to update our inferences about  $\eta_i$  and  $\underline{\theta}_i$ . The posterior distribution of  $\eta_i$  given  $D_i$  is obtained via the standard use of Bayes' theorem

$$p(\eta_i | D_i) \propto \mathcal{L}(\eta_i; D_i) p(\eta_i | D_{i-1}),$$

where  $L(\eta_i; D_i)$  is the likelihood function for  $\eta_i$  given by

$$L(\eta_i; D_i) = \exp\{\eta_i r_i - T_i e^{\eta_i}\}, \quad (4.1)$$

and  $T_i = \sum_{j=1}^{r_i} y_{ij} + (n_i - r_i)\tau_i$  is the *total time on test* at stress level  $S_i$ . It can be shown that

$$(\eta_i | D_i) \sim \mathcal{LG}(a_i + r_i, b_i + T_i), \quad (4.2)$$

implying that  $(\lambda_i | D_i) \sim \mathcal{G}(a_i + r_i, b_i + T_i)$ . The posterior mean and variance of  $\eta_i$  are given by

$$g_i = E[\eta_i | D_i] = \Psi(a_i + r_i) - \log(b_i + T_i), \quad (4.3)$$

$$p_i = \text{Var}[\eta_i | D_i] = \Psi'(a_i + r_i).$$

The next step is to obtain the posterior mean and variance of the state vector  $\underline{\theta}_i$ . We note that the full form of the posterior distribution for  $(\underline{\theta}_i | D_i)$  is not available since the prior of  $(\underline{\theta}_i | D_{i-1})$  is only partially specified. Recognizing the fact that, given  $\eta_i$ , the observation model (2.2) does not depend on  $\underline{\theta}_i$ , WHM developed a method for updating the first two moments of  $(\underline{\theta}_i | D_i)$  using the *linear Bayesian approach* of Hartigan (1969). The details of the method is omitted here and only the main result is presented.

The method described by WHM leads to the updating of the state vector as

$$(\underline{\theta}_i | D_i) \sim [\underline{m}_i, \underline{C}_i], \quad (4.4)$$

where

$$\underline{m}_i = \underline{m}_{i-1} + \underline{s}_i(\underline{g}_i - \underline{f}_i)/q_i, \quad (4.5)$$

$$\underline{C}_i = \underline{R}_i - \underline{s}_i \underline{s}_i' \left\{ \frac{1 - p_i/q_i}{q_i} \right\},$$

with  $\underline{s}_i = \underline{R}_i \underline{F}_i$ . We note that the updating equations for  $\underline{\theta}_i$  in the DGLM setup have the same form as the well-known Kalman filter recursions [see for example Meinhold and Singpurwalla (1983)].

After performing the test at the last stress level  $S_k$ , our aim is to make inference about the life length of an item at use stress conditions  $S_{k+1}$ . In other words, given  $D_k$  we wish to make uncertainty statements about  $Y_{k+1}$  and  $\lambda_{k+1}$  (or equivalently about  $\eta_{k+1}$ ). Given  $D_k$  we have

$$(\eta_k | D_k) \sim \mathcal{LG}(a_k + r_k, b_k + T_k),$$

and

$$(\underline{\theta}_k | D_k) \sim [\underline{m}_k, \underline{C}_k].$$

We describe our uncertainty about the state vector  $\underline{\theta}_{k+1}$  using the system equation; that is,

$$(\underline{\theta}_{k+1} | D_k) \sim [\underline{m}_k, \underline{R}_{k+1}], \quad (4.6)$$

where  $\underline{R}_{k+1} = \underline{C}_k + \underline{W}_{k+1}$ . From the guide relationship of (3.3), we obtain

$f_{k+1} = \underline{F}'_{k+1} \underline{m}_k$  and  $q_{k+1} = \underline{F}'_{k+1} \underline{R}_{k+1} \underline{F}_{k+1}$ . Using the DGLM setup of (3.11), we describe uncertainty about  $\eta_{k+1}$  via the density

$$(\eta_{k+1} | D_k) \sim \mathcal{LG}(a_{k+1}, b_{k+1}), \quad (4.7)$$

where  $a_{k+1}$  and  $b_{k+1}$  are chosen such that  $\Psi(a_{k+1}) - \log(b_{k+1}) = f_{k+1}$  and  $\Psi'(a_{k+1}) = q_{k+1}$ . Thus, inference about the use stress failure rate is made by using the probability density  $(\lambda_{k+1} | D_k) \sim \mathcal{G}(a_{k+1}, b_{k+1})$ .

Inference about the life length of an item operating at use stress conditions is obtained using the predictive density

$$\begin{aligned} p(y_{k+1} | D_k) &= \int_0^{\infty} p(y_{k+1} | \lambda_{k+1}) p(\lambda_{k+1} | D_k) d\lambda_{k+1} \\ &= \frac{a_{k+1}(b_{k+1})^{a_{k+1}}}{(b_{k+1} + y_{k+1})^{a_{k+1}+1}}, \end{aligned} \quad (4.8)$$

which is a Pareto density with reliability function

$$R(y_{k+1} | D_k) = \frac{(b_{k+1})^{a_{k+1}}}{(b_{k+1} + y_{k+1})^{a_{k+1}}}. \quad (4.9)$$

A nice feature of the DGLM setup is that, at any stress level  $S_i$ , we can make inference about the failure behavior at any stress  $S_j < S_i$  (that is,  $i < j$ ) and at the use stress. Assume that we have tested items at stress level  $S_i$  and we have  $(\ell-1)$  stages of testing ahead, that is,  $\ell = k-i+1$ . We

define

$$\hat{\underline{m}}_i(\ell) = E[\underline{\theta}_{i+\ell} | D_i],$$

and

$$\hat{\underline{C}}_i(\ell) = \text{Var}[\underline{\theta}_{i+\ell} | D_i],$$

where  $\hat{\underline{m}}_i(0) = \underline{m}_i$  and  $\hat{\underline{C}}_i(0) = \underline{C}_i$ . Using the system equation of the DGLM setup we obtain

$$\hat{\underline{m}}_i(\ell) = \underline{m}_i, \quad \text{for } \ell \geq 0, \quad (4.10)$$

$$\hat{\underline{C}}_i(\ell) = \hat{\underline{C}}_i(\ell-1) + \underline{W}_{i+\ell} \quad \text{for } \ell > 0.$$

Similarly, using the guide relationship we write

$$\hat{f}_i(\ell) = E[\eta_{i+\ell} | D_i] = \underline{F}'_{i+\ell} \hat{\underline{m}}_i(\ell), \quad (4.11)$$

$$\hat{q}_i(\ell) = \text{Var}[\eta_{i+\ell} | D_i] = \underline{F}'_{i+\ell} \hat{\underline{C}}_i(\ell) \underline{F}_{i+\ell}.$$

Thus, we can use (4.10) and (4.11) in the DGLM setup and make inference about the failure rate at stress levels  $S_j < S_i$  given  $D_i$ .

## 5. EXAMPLE

The method was applied to some accelerated life test data taken from Nelson (1972). This data, given in Table 5.1, represents the times to breakdown of an insulating fluid with subjected to various voltage levels. The insulating fluid is tested at accelerated stress levels 26, 28, 30, 32, 34,

36, and 38 Kv, and inference is to be made concerning the breakdown times for the the insulating fluid at 20 Kv (use stress). In our approach we assume that testing was done in a sequential manner from highest to lowest stress level.

TABLE 5.1

Times to Breakdown of an Insulating Fluid (in Minutes)  
Under Various Values of the Stress

<u>38Kv</u>	<u>36Kv</u>	<u>34Kv</u>	<u>32Kv</u>	<u>30Kv</u>	<u>28Kv</u>	<u>26Kv</u>
.09	.35	.19	.27	7.74	68.85	5.79
.39	.59	.78	.40	17.05	108.29	1579.52
.47	.96	.96	.69	20.46	110.59	2323.70
.73	.99	1.31	.79	21.02	426.07	
.74	1.69	2.78	2.75	22.66	1067.60	
1.13	1.97	3.16	3.91	43.40		
1.40	2.07	4.15	9.88	47.30		
2.38	2.59	4.67	13.95	139.07		
	2.71	4.85	15.93	141.12		
	2.90	6.50	27.80	175.88		
	3.67	7.35	53.24	194.90		
	3.99	8.01	82.85			
	5.35	8.27	89.29			
	13.77	12.06	100.58			
	25.50	31.75	215.10			
		32.52				
		33.91				
		36.71				
		72.89				

Due to our unfamiliarity with the problem at hand, the selection of the prior parameter values is made for illustrative purposes only. We use the Power Law as our time transformation function and specify our prior parameter values as

$$\underline{m}'_0 = \begin{bmatrix} -15.0 & 10.0 \end{bmatrix}, \quad \underline{C}_0 = \begin{bmatrix} 0.5 & 0.0 \\ 0.0 & 0.5 \end{bmatrix}$$

and

$$\underline{W}_i = \begin{bmatrix} 0.01 & 0.0 \\ 0.0 & 0.01 \end{bmatrix}$$

for all stages  $i$ .

Plots of the posterior means of  $\text{Log } \alpha_i$  and  $\beta_i$ , (which are the components of  $\underline{m}_i$ ) for each stage of testing are presented in Figures 5.1 and 5.2 respectively. Note that there is an overall downward trend in both graphs and that after a strong decline from the prior mean values, a more stable behavior is exhibited. Thus

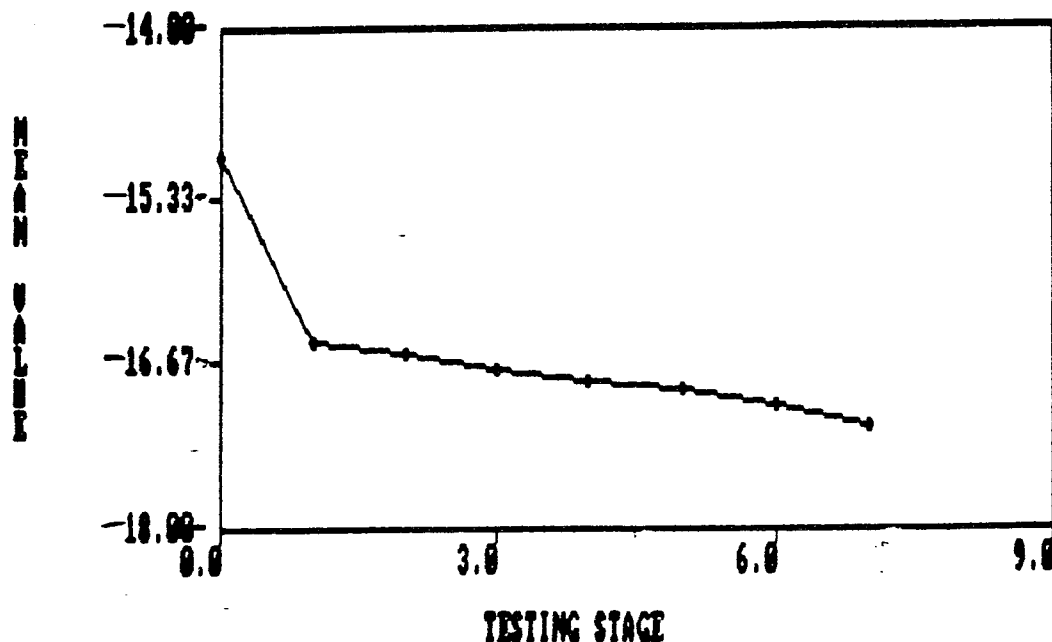


Fig. 5.1 Posterior Means of  $\log \alpha_i$

implying that our prior belief about the effect of stress on failure behavior of the units was stronger than that exhibited by the data.

In Table 5.2 we present the  $E[\lambda_j | D_i]$ ,  $j = i, i+1, \dots, k, k+1$  for each testing stage  $i$ . Columns of this table exhibit our assessment of the failure behavior of the system (insulating fluid) under the different levels of stress based on the total

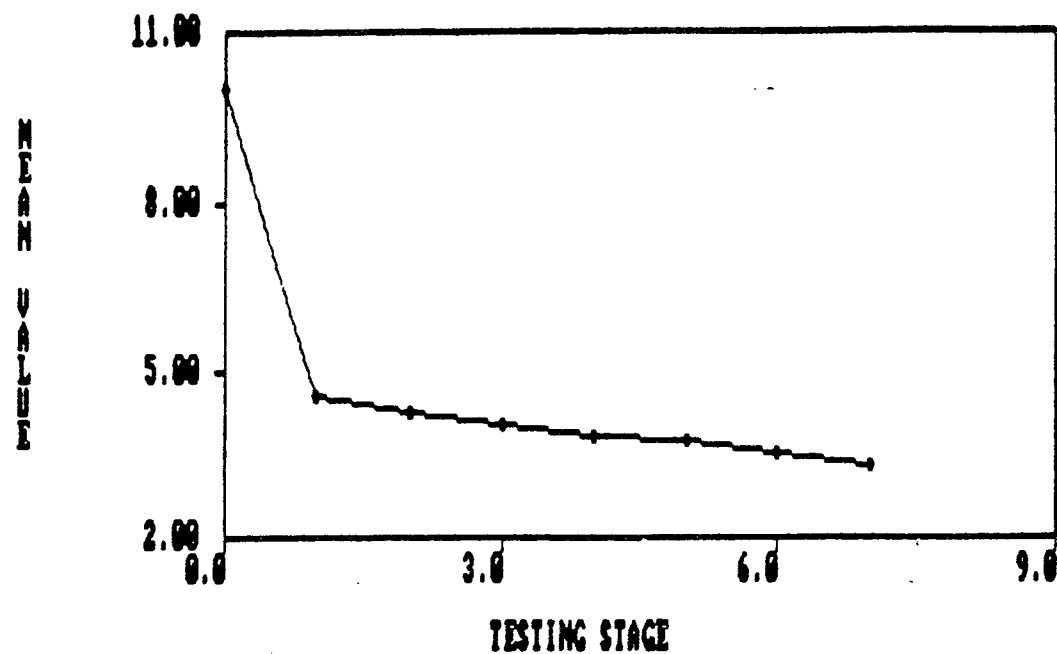


Fig. 5.2 Posterior Means of  $\beta_i$

amount of information available after each stage of testing. In Figure 5.3 we plot the last row of this table to illustrate the evolution of the failure rate predictions for use stress. In addition, in Figure 5.4 we illustrate the predictive reliability functions for use stress after testing stages 1, 3, 5, and 7.

TABLE 5.2

Failure Rate Predictions at Different Testing Stages

PREDICTION FOR STAGE	TESTING STAGE						
	1	2	3	4	5	6	7
1	1.1102						
2	0.8155	0.2546					
3	0.6288	0.1944	0.0810				
4	0.4773	0.1503	0.0622	0.0293			
5	0.3559	0.1142	0.0480	0.0234	0.0154		
6	0.2601	0.0852	0.0363	0.0171	0.0115	0.0047	
7	0.1857	0.0622	0.0270	0.0129	0.0087	0.0034	0.0015
8 (use stress)	0.0563	0.0204	0.0094	0.0047	0.0033	0.0014	0.0006

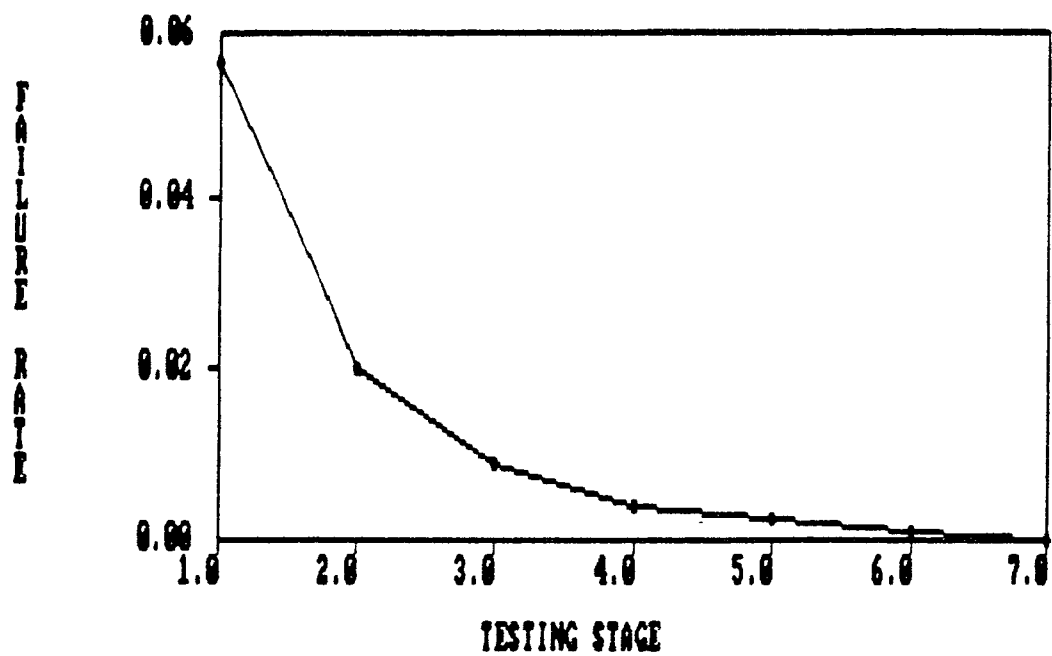


Fig. 5.3 Failure Rate Predictions for Use Stress at Different Testing Stages

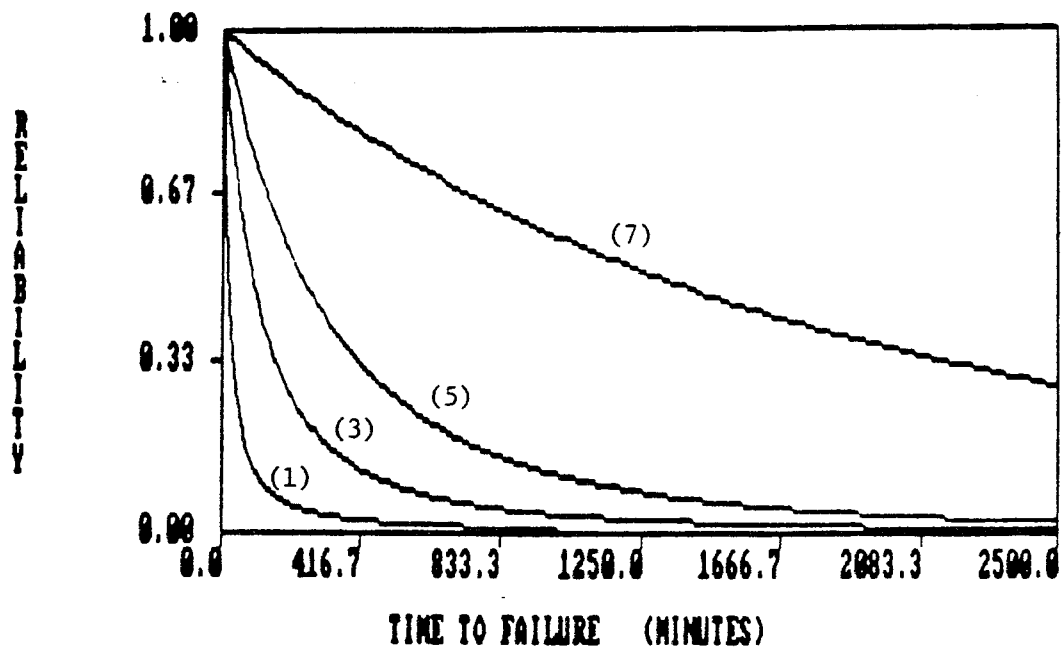


Fig. 5.4 Predictive Reliability Function for Use Stress at Testing Stages  $i = 1, 3, 5,$  and  $7$

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